Solutions to Mock JEE MAIN – 5 (CBT) | JEE 2024

PHYSICS

SECTION-1

1.(C) Given, $E = 30\cos(kz - 5\times10^8 t)$ … (i)

We know that $E = E_0\cos(kz - \omega t)$ … (ii)

Comparing the eqs. (i) and (ii), we get $\omega = 5\times10^8\ rad/s$

But we also know that $k = \frac{2\pi}{\lambda}$ and $\omega = 2\pi v$

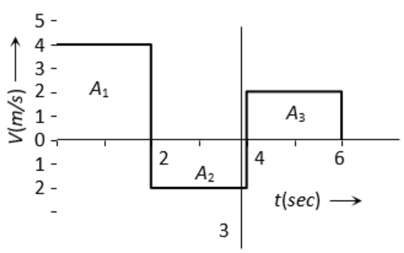
Where, λ is wavelength and v is frequency of the wave.

$\therefore \quad \frac{\omega}{k} = \frac{2\pi v}{2\pi/\lambda} = v\lambda = c \quad \Rightarrow \quad c = \frac{\omega}{k}$

or $\quad k = \frac{\omega}{c} = \frac{5\times10^8}{3\times10^8} \quad \Rightarrow \quad k = \frac{5}{3} = 1.66\ rad/m$

2.(A) Displacement = Summation of all the area with sign

$= (A_1) + (-A_2) + (A_3) = (2\times4) + (-2\times2) + (2\times2)$



$\therefore$ Displacement = 8 m

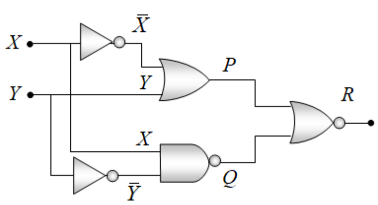
Distance = Summation of all the areas without sign

$= |A_1| + |-A_2| + |A_3| = |8| + |-4| + |4| = 8 + 4 + 4 \quad \therefore \quad$ Distance = 16 m.

3.(A) $\frac{g_1}{g_2} = \frac{\rho_1}{\rho_2}\times\frac{R_1}{R_2} = \frac{3}{2}\times\frac{2}{3} = 1$

4.(C) $v = \sqrt{\mu g r} = \sqrt{0.5\times9.8\times40} = \sqrt{196} = 14\ m/s$

5.(C)



The truth table can be written as

X	Y	$\bar{X}$	$\bar{Y}$	$P = \bar{X} + Y$	$Q = \overline{X.\bar{Y}}$	$R = \overline{P + Q}$
0	1	1	0	1	1	0
1	1	0	0	1	1	0
1	0	0	1	0	0	1
0	0	1	1	1	1	0

Hence X = 1, Y = 0 gives output R =1

6.(A) $2\times\frac{1}{2}\left(\frac{\varepsilon_0\ell w}{d}\right)v^2=\frac{1}{2}\left[\frac{k\varepsilon_0 xw}{d}+\frac{\varepsilon_0(\ell-x)w}{d}\right]v^2$, here $k=4$; $x=\frac{l}{3}$ here $\varepsilon_r=4$

7.(A) Resistance R_1 of 500 W bulb $=\frac{(220)^2}{500}$

Resistance R_2 of 200 W bulb $=\frac{(220)^2}{200}$

When joined in parallel, the potential difference across both the bulbs will be same.

Ratio of heat produced $=\frac{V^2/R_1}{V^2/R_2}=\frac{R_2}{R_1}=\frac{5}{2}$

When joined in series, the same current will flow through both the bulbs.

Ratio of heat produced $=\frac{i^2R_1}{i^2R_2}=\frac{R_1}{R_2}=\frac{2}{5}$

8.(B) Induced emf is given by $e=-\frac{d\phi}{dt}$

If the radius of loop is r at a time t, then the instantaneous magnetic flux is given by

$$\phi=\pi r^2 B$$

$$\therefore \quad e=-\frac{d}{dt}(\pi r^2 B)=-\pi B\left(\frac{2r\,dr}{dt}\right)=-2\pi Br\frac{dr}{dt}$$

Numerically, $e=2\pi Br\left(\frac{dr}{dt}\right)$

9.(C) $(ms\Delta\theta)+mL=i_{rms}^2Rt \quad\Rightarrow\quad t=\left(\frac{ms\Delta\theta+mL}{i_{rms}^2Rt}\right)$; $\quad t\approx 22$ minutes

10.(D) For adiabatic process $\Delta W=-\Delta U \qquad (\because \Delta Q=0)$

$\Rightarrow \quad \Delta W=-(-50)=+50J$

11.(B) $\mu\propto\frac{1}{v} \quad\Rightarrow\quad \frac{\mu_g}{\mu_w}=\frac{v_w}{v_g}\Rightarrow\frac{3/2}{4/3}=\frac{v_w}{2\times10^8} \quad\Rightarrow\quad v_w=2.25\times10^8\,m/s$

12.(C) $W_0=\frac{hc}{\lambda_0}=\frac{6.625\times10^{-34}\times3\times10^8}{5000\times10^{-10}}J=4\times10^{-19}J$

13.(C) We can write $E=E.\hat{i}$ and $B=B\hat{k}$ Velocity of the particle will be along $q.\,E$ direction. Therefore, we can write $v=Aq\,E\hat{i}$ In E, B and v, A E and B are positive constants while q can be positive or negative. Now, magnetic force on the particle will be $F_m=q(v\times B)=q\{AqE\hat{i}\}\times\{B\hat{k}\} \;=q^2AEB(\hat{i}\times\hat{k})$

$F_m=q^2AEB(-\hat{j})$

Since, F_m is along negative y-axis, no matter what is the sign of charge q. Therefore, all ions will deflect towards negative y-direction.

14.(A) $\sigma_i=\frac{\theta}{i}=\frac{\theta}{iG}\cdot G=\sigma_V G \quad\Rightarrow\quad \frac{\sigma_i}{G}=\sigma_V$

15.(A) In S.H.M. when acceleration is negative maximum or positive maximum, the velocity is zero so kinetic energy is also zero. Similarly for zero acceleration, velocity is maximum so kinetic energy is also maximum.

16.(B) $Q = m.c.\Delta\theta \quad \Rightarrow \quad c = \dfrac{Q}{m.\Delta\theta}$

In temperature measurement scale $\Delta T°F > \Delta T°C$ so $S_{°F} < S_{°C}$.

17.(B) Statement 1 is correct while statement 2 is incorrect.

18.(B) $\tau = \dfrac{\text{mean free path}}{V_{\text{rms}}} = \dfrac{1}{\sqrt{2}n\pi d^2}\dfrac{1}{V_{\text{rms}}}$

As $n = \dfrac{N}{V}$ & $V_{\text{rms}} = \sqrt{\dfrac{3RT}{M}} \quad \therefore \quad \tau \propto \dfrac{V}{\sqrt{T}}$

Also $TV^{\gamma-1} = \text{constant} \Rightarrow T \propto V^{1-\gamma}$

$$\therefore \quad \tau \propto \frac{V}{V^{\frac{1-\gamma}{2}}} \propto V^{\frac{\gamma+1}{2}}; \quad \frac{\tau_2}{\tau_1} = \left(\frac{V_2}{V_1}\right)^{\frac{\gamma+1}{2}} = 2^{\frac{\gamma+1}{2}}$$

19.(D) $u = 250\,m/s$, $v = 0$, $s = 0.12\,metre$

$$F = ma = m\left(\frac{u^2 - v^2}{2s}\right) = \frac{20\times10^{-3}\times(250)^2}{2\times0.12}$$

$$\therefore \quad F = 5.2\times10^3\ N$$

20.(B) For photon: $E = hv$ or $E = \dfrac{hc}{\lambda} \quad \Rightarrow \quad \lambda_2 = \dfrac{hc}{E}$ … (i)

For proton: $E = \dfrac{1}{2}m_p v_p^2$; $\quad E = \dfrac{1}{2}\dfrac{m_p^2 v_p^2}{m_p} \quad \Rightarrow \quad P = \sqrt{2mE}$

Form de Broglie equation, $P = \dfrac{h}{\lambda_1}$

$$\Rightarrow \quad \lambda_1 = \frac{h}{P} = \frac{h}{\sqrt{2mE}} \quad \text{... (ii)}$$

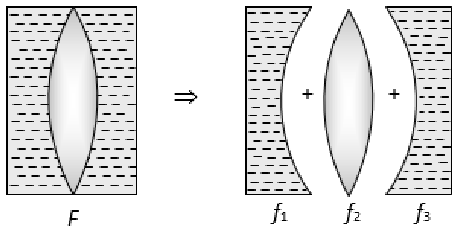
$$\frac{\lambda_2}{\lambda_1} = \frac{hc}{E\times\frac{h}{\sqrt{2mE}}} \propto E^{-1/2}$$

SECTION-2

1.(9) $\dfrac{1}{\lambda} = R\left(\dfrac{1}{2^2} - \dfrac{1}{4^2}\right) = \dfrac{3R}{16} \Rightarrow \lambda = \dfrac{16}{3R} = \dfrac{16}{3}\times10^{-5}\,cm$

Frequency $n = \dfrac{c}{\lambda} = \dfrac{3\times10^{10}}{\frac{16}{3}\times10^{-5}} = \dfrac{9}{16}\times10^{15}\,Hz$

2.(100) $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}$



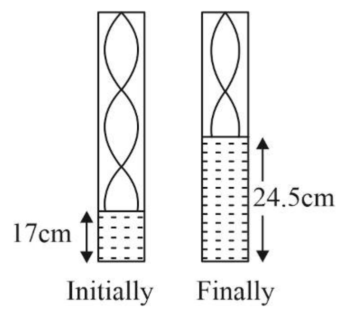
$$\frac{1}{f_1}=(1.6-1)\left(\frac{1}{\infty}-\frac{1}{20}\right)=-\frac{0.6}{20}=-\frac{3}{100} \quad \ldots \text{(i)}$$

$$\frac{1}{f_2}=(1.5-1)\left(\frac{1}{20}-\frac{1}{-20}\right)=\frac{1}{20} \quad \ldots \text{(ii)}$$

$$\frac{1}{f_3}=(1.6-1)\left(\frac{1}{-20}-\frac{1}{\infty}\right)=-\frac{3}{100} \quad \ldots \text{(iii)}$$

$$\Rightarrow \quad \frac{1}{F}=-\frac{3}{100}+\frac{1}{20}-\frac{3}{100} \Rightarrow F=-100cm$$

3.(2200) From initial & final heights of water level,



$$\frac{\lambda}{2}=24.5-17$$

$$\lambda=15cm$$

$$\Rightarrow \quad \frac{v}{\upsilon}=15\times10^{-2}$$

$$\Rightarrow \quad \upsilon=\frac{330}{15\times10^{-2}}$$

$$\Rightarrow \quad \upsilon=2200\text{Hz}$$

4.(7) $W=\vec{F}.\vec{s}=(5\hat{i}+3\hat{j}).(2\hat{i}-\hat{j})=10-3=7J$

5.(64) Given for disc of radius 'R' $\quad \sigma=\sigma_0(1-r/R)$

If total charge on disc is Q

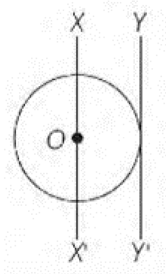
For large spherical surface by gauss law $\phi_0=\frac{Q}{\epsilon_0}$.

If charge on disc upto radius $R/4$ is q by Gauss Law $\phi=\frac{q}{\epsilon_0}$

$$\Rightarrow \quad \frac{\phi_0}{\phi}=\frac{Q}{q}=\frac{\int_0^R 2\pi r dr\sigma}{\int_0^{R/4} 2\pi r dr\sigma} \quad =\frac{\int_0^R\left(r-\frac{r^2}{R}\right)dr}{\int_0^{R/4}\left(r-\frac{r^2}{R}\right)dr}=\frac{R^2/6}{R^2\left(\frac{5}{64\times3}\right)}$$

$$\frac{\phi_0}{\phi}=\frac{64\times3}{5\times6}=\frac{32}{5}=6.4\,; \qquad \boxed{\frac{\phi_0}{\phi}=6.4}$$

6.(10) Moment of inertia about the given axis i.e., about YY',



$$I_{YY'} = I_{XX'} + MR^2 = 2 + 2\times(2)^2 = 2 + 8 = 10 kg\text{-}m^2$$

7.(16) strain $\propto$ stress $\propto \frac{\text{F}}{\text{A}}$

Ratio of strain $= \frac{A_2}{A_1} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{4}{1}\right)^2 = \frac{16}{1}$

8.(2) $|dq| = \frac{d\varphi}{R} = i\ dt =$ Area under $i-t$ graph

$\therefore \quad d\varphi =$ (Area under $i-t$ graph) R

$= \frac{1}{2}\times 4\times 0.1\times(10) = 2\ Wb.$

9.(200) $R = \frac{u^2 \sin 2\theta}{g} = R \propto u^2.$

So if the speed of projection doubled, the range will becomes four times,

i.e., $\quad 4\times 50 = 200\,m$

10.(2) Equivalent external resistance of the given circuit $R_{eq} = 4\Omega$

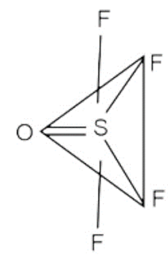
Current given by the cell $i = \frac{E}{R_{eq}+r} = \frac{10}{(4+1)} = 2\,A$

Hence, $(V_A - V_B) = \frac{i}{2}\times(R_2 - R_1) = \frac{2}{2}(2-4) = -2V.$

CHEMISTRY

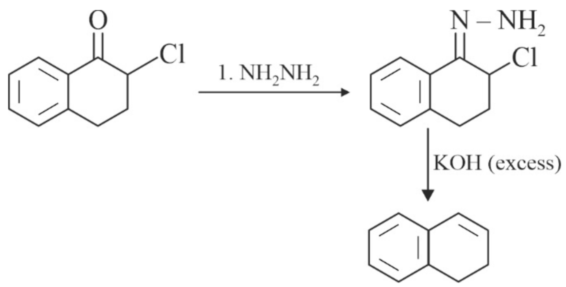
SECTION-1

1.(A) Geometry of SOF_4 is trigonal bipyramidal having $S=O$ on trigonal plane. Double bonds occupy more space. Hence occupy equatorial position.



2.(D) As the temperature increases λ_{max} decreases. As the temperature is increased, wavelength at which intensity is maximum, decreases and hence energy increases. Also, a blackbody is in thermal equilibrium with its surroundings.

3.(D)



Wolff-kishner reduction and elimination of HCl

4.(A) Aryl halide & vinyl halide do not undergo S_N2.

5.(D) $K[MnO_4]$ $\Rightarrow$ Complex is anion so manganese will be manganate.

Potassium tetraoxidomanganate (VII)

6.(A) Order of screening effect or shielding effect is

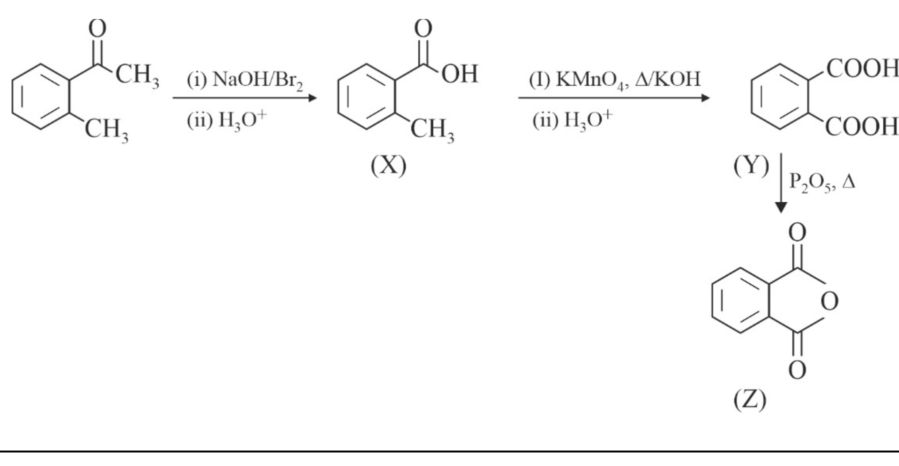
$s > p > d > f$

7.(A) V_2O_5 & Cr_2O_3 amphoteric oxides

$Mn_2O_7, CrO_3 \rightarrow$ Acidic

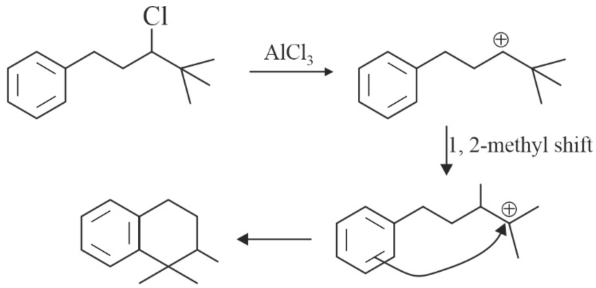
$CrO, V_2O_4 \rightarrow$ Basic

8.(D)



9.(A) Isomer-1 is fac isomer
Isomer-2 is mer isomer
Not all $Cl-Co-Cl$ & $N-Co-N$ bond angles are 90° in isomer-2
As Cl is inside the coordination sphere, so it is not free.
Only one $Cl-Co-Cl$ angle is 180° in isomer-2.

10.(A)



11.(C) Complementary of A is T
Complementary of G is C
5'-G-A-A-T-T-C-3' ; 3'-C-T-T-A-A-G-5'

12.(C) Actinoids form relatively more stable complexes as compared to lanthanoids because of actinoids can utilise their 5f orbitals along with 6d orbitals in bonding but lanthanoids do not use their 4f orbitals for bonding.

13.(A) In cell I, reactions are:

At cathode: $Cu^{2+} + 2e^- \rightarrow Cu(s)$

At Anode: $Cu(s) \rightarrow Cu^{2+} + 2e^-$

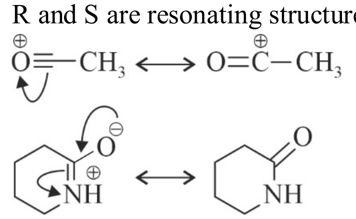
In cell II, reactions are:

At cathode: $Cu^{2+} + 2e^- \rightarrow Cu(s)$

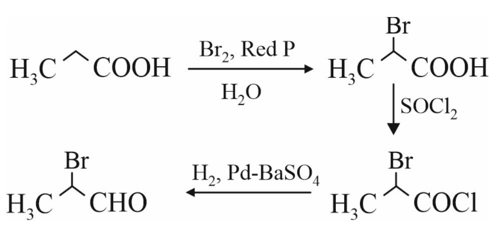
At anode: $2H_2O \rightarrow O_2 + 4H^+ + 4e^-$

So pH remains same in I and decreases in II.

14.(D) P and Q are not resonating structures
R and S are resonating structures



15.(A)

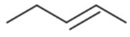


16.(D) At melting point $\Delta G = 0$

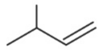
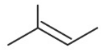
ΔG decides the spontaneity of the reaction not ΔG^0, if ΔG^0 is positive, K is less than 1.

17.(B) p-p single bond is stronger than N-N
Metalloids only exists in p-block.
$d\pi - p\pi$ bonding is strong for heavier elements not $p\pi - p\pi$ bonding.

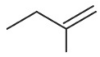
18.(C) Alkene B will show cis-trans isomerism so total alkenes is 6

 +  +

A B

 + 

C D



E

19.(A) Rate $= k[P]^x[Q]^y$

In entry 1 & 2 concentration of P is very large than Q so we can neglect P.

Rate $= = k'[Q]^y$ for entry 1 & 2

Now as $t_{1/2}$ gets double when concentration of Q gets double i.e. zero order $\Rightarrow$ $y = 0$

In entry 3 & 4 concentration of Q is very large than P so we can neglect Q.

Rate $= = k''[P]^x$

Now as $t_{1/2}$ remains same when concentration of P gets double i.e. 1st order $\Rightarrow$ $x = 1$

So finally, Rate $= [P]$

Rate $= -\frac{dP}{dt} = -\frac{dQ}{dt} = \frac{dR}{dt} = k[P]$

20.(C) $3Br_2 + 6OH^- \longrightarrow 5Br^- + BrO_3^- + 3H_2O$ number of moles of e^- transferred = 5

For three Br_2 five electrons are transferred hence for one Br_2 5/3 electrons are transferred

n-factor of $Br_2 = 5/3$

Equivalent weight of $Br_2 = \frac{M}{5/3} = \frac{3M}{5}$

SECTION-2

1.(1) $E = E^0 - \frac{0.059}{n}\log_{10} Q$ at 25°C

At equilibrium, $E = 0, Q = K$

$0 = E^0 - \frac{0.059}{n}\log_{10} K$ or, $K = \text{antilog}\left[\frac{nE^0}{0.059}\right]$

or, $K = \text{antilog}\left[\frac{2 \times 0.295}{0.059}\right] = \text{antilog}\left[\frac{0.590}{0.059}\right] = \text{antilog}\,10 = 1 \times 10^{10}$

2.(6) There are six α-hydrogens present and thus there will be six contributing structures showing hyperconjugation (involving C – H bonds).

3.(96)

$$P \rightleftharpoons Q \qquad K = 1.5$$

$t = 0$ 2

$t = t_{eq}$ $2 - y$ y

$\frac{y}{2-y} = \frac{3}{2} \Rightarrow 2y = 6 - 3y \Rightarrow y = \frac{6}{5}$

Now $P \rightleftharpoons Q$

$t = 0 \quad \frac{4}{5}\left(1-\frac{1}{2}\right) \qquad \frac{6}{5}$

$t = 0 \quad \frac{2}{5} \qquad \frac{6}{5}$

$t = t_{eq} \quad \frac{2}{5}+t \qquad \frac{6}{5}-t$ (As half the amount of P is removed so reaction will go backward)

$$\frac{\left(\frac{6}{5}-t\right)}{\left(\frac{2}{5}+t\right)} = \frac{3}{2} \Rightarrow \frac{12}{5}-2t = \frac{6}{5}+3t \qquad \Rightarrow \qquad 5t = \frac{6}{5} \Rightarrow t = \frac{6}{25}$$

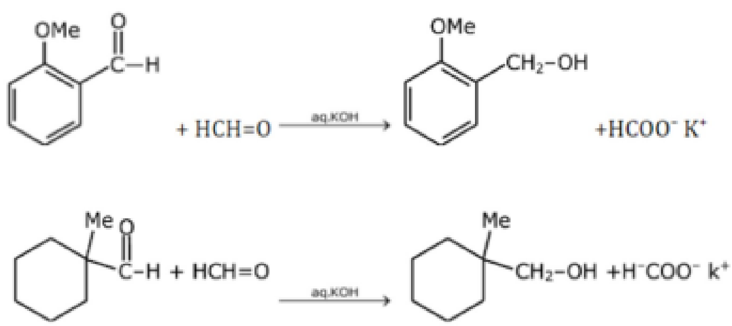
$$[Q] = \frac{6}{5}-\frac{6}{25} = \frac{6}{5}\left\{1-\frac{1}{5}\right\} = \frac{6}{5}\times\frac{4}{5} \qquad = \frac{24}{25} = 0.96$$

4.(30) Haber's process, $N_2 + 3H_2 \rightarrow 2NH_3$

2 moles of NH_3 are formed by 3 moles of H_2

$\therefore$ 20 moles of NH_3 will be formed by 30 moles of H_2

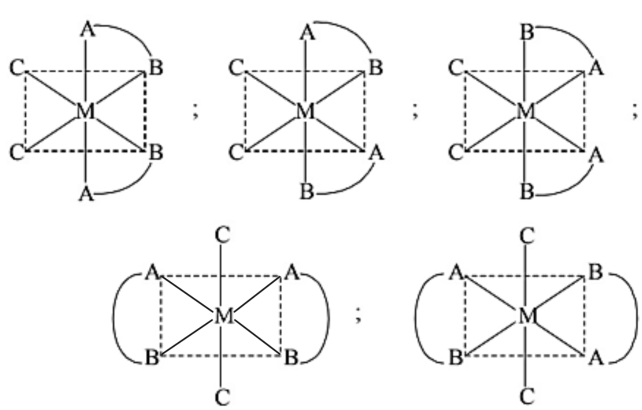
5.(2)



These are examples of Cannizzaro reactions.
Aldehyde without α-H gives a Cannizzaro reaction and forms alcohol and salt of carboxylic acid.

6.(1) With respect to P, the reaction is first order as the successive $t_{1/2}$ is independent of its initial concentration. With respect to Q order is zero.

7.(5) The given complex is of the type $[M(AB)_2C_2]$. Its possible geometrical isomers are:



There are 5 geometrical isomers.

8.(600) For overall cycle: $\Delta U = 0$

$\therefore \quad q = -W$

$$q_{AB} + q_{BC} + q_{CA} = -[W_{AB} + W_{BC} + W_{CA}]$$

$$-800 + 0 + 100 = -[800 + 500 + W_{CA}]$$

$$W_{CA} = -600J$$

9.(75) The dissociation of K_2SO_4 may be written as

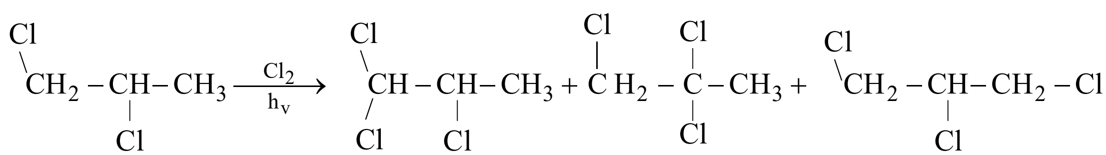
$$\underset{c(1-\alpha)}{K_2SO_4} \rightleftharpoons \underset{c(2\alpha)}{2K^+} + \underset{c\alpha}{SO_4^{2-}}$$

The concentration of species in the solution is $c(1-\alpha) + c(2\alpha) + c\alpha = c(1+2\alpha)$ (where c = 0.004 M).

Hence, $c(1+2\alpha) = 0.01M$ or $\alpha = \frac{1}{2}\left(\frac{0.01}{0.004} - 1\right) = 0.75$

The percent of dissociation is 0.75×100, i.e. 75%

10.(3)



MATHEMATICS

SECTION-1

1.(B) $\because \quad A=\int_0^{\pi}\frac{\cos x}{(x+2)^2}dx \quad \Rightarrow \quad A=\left\{\cos x\left(\frac{-1}{x+2}\right)\right\}_0^{\pi}-\int_0^{\pi}(-\sin x)\left(\frac{-1}{x+2}\right)dx$

$\Rightarrow \quad A=\frac{1}{\pi+2}+\frac{1}{2}-\int_0^{\pi}\frac{\sin x}{x+2}dx$

$A=\frac{1}{\pi+2}+\frac{1}{2}-\int_0^{\pi/2}\frac{\sin 2x}{x+1}dx\,;\quad \int_0^{\pi/2}\frac{\sin 2x}{x+1}dx=\frac{1}{\pi+2}+\frac{1}{2}-A$

2.(D) $\vec{a}=\alpha\hat{i}+\hat{j}+\beta\hat{k}\,;\qquad \vec{b}=3\hat{i}-5\hat{j}+4\hat{k}$

$\vec{a}\times\vec{b}=\hat{i}(4+5\beta)-\hat{j}(4\alpha-3\beta)+k(-5\alpha-3)$

Given $\vec{a}\times\vec{b}=-\hat{i}+9\hat{j}+12\hat{k}$

By comparison

$4+5\beta=-1 \qquad -5\alpha-3=12$

$5\beta=-5 \qquad -5\alpha=15$

$\boxed{\beta=-1} \qquad \boxed{\alpha=-3}$

$\vec{a}=-3\hat{i}+\hat{j}-\hat{k}\,;\qquad \vec{b}=3\hat{i}-5\hat{j}+4\hat{k}$

$\vec{b}-2\vec{a}=(3+6)\hat{i}-7\hat{j}+6\hat{k} \qquad =9\hat{i}-7\hat{j}+6\hat{k}$

$\vec{b}+\vec{a}=0\hat{i}-4\hat{j}+3\hat{k}$

Projection of $(\vec{b}-2\vec{a})$ on $(\vec{b}+\vec{a})$

$=\frac{(\vec{b}+\vec{a})\cdot(\vec{b}-2\vec{a})}{(\vec{b}+\vec{a})} \qquad =\frac{28+18}{5}=\frac{46}{5}$

3.(B) $a^2=\frac{1}{4},\ b^2=\frac{1}{9}$ let P be $(x_1,\ y_1)$

Equation normal $\frac{1x}{4x_1}-\frac{1y}{9y_1}=\frac{5}{36}$

It cuts y-axis at $\left(0,-\frac{5y_1}{4}\right)\Rightarrow\frac{-5y_1}{4}=\frac{-5}{8\sqrt{3}}\Rightarrow y_1=\frac{1}{2\sqrt{3}} \quad \therefore \quad x_1=\frac{1}{4}$

Thus $\alpha=\frac{5}{36}$; Area of $\Delta OAB=\frac{1}{2}\times\frac{5}{36}\times\frac{5}{8\sqrt{3}}=\frac{25}{576\sqrt{3}}$

4.(A)

A	B
$\bar{x}_1=40$	$\bar{x}_2=55$
$\sigma_1=\alpha$	$\sigma_2=30-\alpha$
$n_1=100$	$n_2=n$
$A+B$;	$\bar{x}=50$
$\sigma^2=350$;	$100+n$

$\bar{x} = \dfrac{100 \times 40 + 55n}{100 + n}$

$5000 + 50n = 4000 + 55n$; $1000 = 5n$

$n = 200$; $\sigma_1^2 = \dfrac{\sum x_i^2}{100} - 40^2$; $\sigma_2^2 = \dfrac{\sum x_j^2}{100} - 55^2$

$350 = \sigma^2 = \dfrac{\sum x_i^2 + \sum x_i^2}{300} - (\bar{x})^2$; $350 = \dfrac{(1600 + \alpha^2) \times 100 + [(30 - \alpha)^2 + 3025] \times 200}{300} - (50)^2$

$2850 \times 3 = \alpha^2 + 2(30 - \alpha)^2 + 1600 + 6050$; $8550 = \alpha^2 + 2(30 - \alpha)^2 + 7650$

$\alpha^2 + 2(30 - \alpha)^2 = 900$; $\alpha^2 - 40\alpha + 300 = 0$

$\alpha = 10, 30$; $\sigma_1^2 + \sigma_2^2 = 10^2 + 20^2 = 500$

5.(D) (A) Function is continuous on (0, 2)

(B) Not differentiable at $\{-1, 0, 1\}$

(C) $f(-1) = f(1)$

6.(A) $x_1 + x_2 + x_3 + x_4 = 15$; $0 \le x_1 \le 5,\ x_2 \ge 2$

Coefficient of x^{15} in $(1 + x + x^2 + ... + x^5)(x^2 + x^3 + ...)(1 + x + x^2 + ...)^2$; $\dfrac{x^2(1 - x^6)}{(1 - x)^4}$

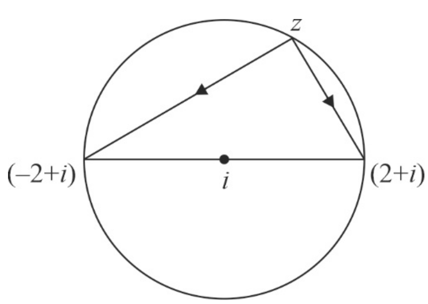
Coefficient of x^{13} in $(1 - x^6)(1 - x)^{-4}$

$(1 - x)^{-4} - x^6(1 - x)^{-4}$; ${}^{4+13-1}C_{13} - {}^{4+7-1}C_7$; ${}^{16}C_{13} - {}^{10}C_7 = 440$

7.(C) $|\overrightarrow{op}| = M = \sqrt{\cos^2 t + \sin^2 t + 2\cos t \sin t(\hat{a}.\hat{b})} = \sqrt{1 + (\sin 2t)(\hat{a}.\hat{b})}$

$(M)_{\max} = \sqrt{1 + (\hat{a}.\hat{b})}$, when $t = \dfrac{\pi}{4}$; $\hat{\mu} = \dfrac{\overrightarrow{op}}{|\overrightarrow{op}|} = \dfrac{\frac{1}{\sqrt{2}}(\hat{a} + \hat{b})}{\frac{1}{\sqrt{2}}|\hat{a} + \hat{b}|}$

8.(A)



9.(D) $E_1 =$ people with membership in only 1 club.

$E_2 =$ people with membership in exactly 2 clubs.

$E_3 =$ people with membership in 3 clubs.

$E_1 = 70$ $E_3 = 10$

$A \cup B \cup C = E_1 + E_2 + E_3$

$\Rightarrow \quad A \cup B \cup C = 80 + E_2$ … (i)

$A \cup B \cup C = A + B + C - \sum(A \cap B) + A \cap B \cap C$

$\Rightarrow \quad A \cup B \cup C = 40 + 50 + 60 - \sum(A \cap B) + A \cap B \cap C$

$\Rightarrow \quad A \cup B \cup C = 40 + 50 + 60 - \sum(A \cap B) + 10$

$\Rightarrow \quad A \cup B \cup C = 160 - \sum(A \cap B)$ …(ii)

Also, $E_2 = \sum(A \cap B) - 3(A \cap B \cap C)$ …(iii)

$= \sum(A \cap B) - 30$

From (i), (ii) and (iii)

$160 - \sum(A \cap B) = 80 + \sum(A \cap B) - 30$

$\Rightarrow \quad 160 - \sum(A \cap B) = 50 + \sum(A \cap B) \quad \Rightarrow \quad \sum(A \cap B) = 55$

$\Rightarrow \quad E_2 = 25 \quad \Rightarrow \quad A \cup B \cup C = 160 - 55 = 105 \quad \Rightarrow \quad P(E_2) = \frac{n(E_2)}{n(S)} = \frac{25}{105} = \frac{5}{21}$

10.(A) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}; a, b, c, d \in \{0, 1, 2, 3, 4, 5\}$

$a + b + c + d = p, p \in \{3, 5, 7\}$

Case (i)

$a + b + c + d = 3; a, b, c, d \in \{0, 1, 2, 3\}$

No. of ways $= {}^{3+4-1}C_{4-1} = {}^6C_3 = 20$ … (1)

Case (ii)

$a + b + c + d = 5; a, b, c, d \in \{0, 1, 2, 3, 4, 5\}$

No. of ways $= {}^{5+4-1}C_{4-1} = {}^8C_3 = 56$ … (2)

Case (iii)

$a + b + c + d = 7$

Number of ways = total ways when $a, b, c, d \in \{0, 1, 2, 3, 4, 5, 6, 7\}$ – total ways when $a, b, c, d \not\subset \{6, 7\}$

No of ways $= {}^{7+4-1}C_{4-1} - \left(\frac{\lfloor 4}{\lfloor 3} + \frac{\lfloor 4}{\lfloor 2}\right)$

$= {}^{10}C_3 - 16 = 104$ … (3)

Hence total no. of ways = 180

11.(D) Circle passes through the foci of ellipse with radius = 5
Hence PQ = diameter of circle = 2 × 5 = 10

12.(A) $\log_2 x = t$ (say)

$t^4 - (2 - 5t)^2 - 20t + 148 < 0;$ $\qquad t^4 - 4 - 25t^2 + 20t - 20t + 148 < 0$

$t^4 - 25t^2 + 144 < 0;$ $\qquad (t^2 - 16)(t^2 - 9) < 0$

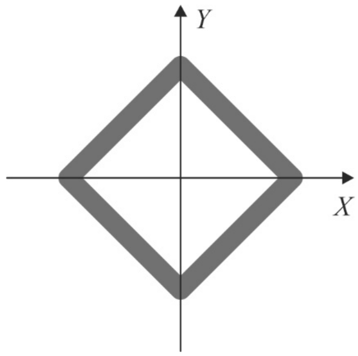
$(t + 4)(t - 4)(t + 3)(t - 3) < 0$

$t \in (-4,-3) \cup (3,4)$

$-4 < \log_2 x < -3$ or $3 < \log_2 x < 4$

$\frac{1}{16} < x < \frac{1}{8}$ or $8 < x < 16$; $x \in \left(\frac{1}{16}, \frac{1}{8}\right) \cup (8,16)$

13.(C) Put $x-2 = X, y+1 = Y$



Then the required region is defined by

$1 \le |X| + |Y| \le 2$

Required area $= (2\sqrt{2})^2 - (\sqrt{2})^2 = 8 - 2 = 6$

14.(D) Given $xRy \Leftrightarrow \sin^2 x + \cos^2 y = 1$

Now $\sin^2 x + \cos^2 x = 1$, So R is Reflexive. i.e., xRx

Let $xRy \Rightarrow \sin^2 x + \cos^2 y = 1$

$\Rightarrow 1 - \cos^2 x + 1 - \sin^2 y = 1 \Rightarrow \sin^2 y + \cos^2 x = 1$

So $xRy \Rightarrow yRx \therefore$ R is symmetric

Now let xRy and yRz holds

i.e., $\sin^2 x + \cos^2 y = 1$ and $\sin^2 y + \cos^2 z = 1$

So $\sin^2 x + \cos^2 z = 1$ (from above two equations)

So xRy & $yRz \Rightarrow xRz \therefore$ R is Transitive

Hence R is an equivalence relation

15.(A) $3x^2y^2 + \cos(xy) - xy\sin(xy) + \frac{dy}{dx}\{2x^3y - x^2\sin(xy)\} = 0$ Can be written as

$3x^2y^2dx + \cos(xy)dx - xy\sin(xy)dx + 2x^3ydy - x^2\sin(xy)dy = 0$

$\Rightarrow d(x^3y^2) + d(x\cos(xy)) = 0$

By integrating we get

$x^3y^2 + x\cos(xy) = k$

Aliter :

Let $xy = t \Rightarrow y = \frac{t}{x}$ $\Rightarrow \frac{dy}{dx} = \frac{1}{x}\frac{dt}{dx} - \frac{t}{x^2}$

$\Rightarrow 3x^2y^2 + \cos(xy) - xy\sin(xy) + x^2\frac{dy}{dx}(2xy - \sin(xy)) = 0$

$\Rightarrow \quad 3t^2 + \cos t - t\sin t + x^2\left(\frac{1}{x}\frac{dt}{dx} - \frac{t}{x^2}\right)(2t - \sin t) = 0$

$\Rightarrow \quad 3t^2 + \cos t - t\sin t + \left(x\frac{dt}{dx} - t\right)(2t - \sin t) = 0$

$\Rightarrow \quad x\frac{dt}{dx} - t + \frac{t^2 + \cos t + 2t^2 - t\sin t}{2t - \sin t} = 0$

$\Rightarrow \quad x\frac{dt}{dx} - t + \frac{t^2 + \cos t}{2t - \sin t} + t = 0 \quad \Rightarrow \quad x\frac{dt}{dx} + \frac{t^2 + \cos t}{2t - \sin t} = 0$

$\Rightarrow \quad \int \frac{(2t - \sin t)}{t^2 + \cos t}dt + \int \frac{dx}{x} = 0 \quad \Rightarrow \quad \ell n(t^2 + \cos t) + \ell nx = k_1$

$\Rightarrow \quad x(t^2 + \cos t) = k \quad \Rightarrow \quad x(x^2y^2 + \cos(xy)) = k; k \in R$

16.(A) $A^T + Adj\, B = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ … (i)

$A + (Adj\, B^T) = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}, A^T - Adj\, B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ … (ii)

So (i) + (ii), $2A^T = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} \quad \therefore \quad A^T = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \Rightarrow A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

Also, $A + Adj(B^T) = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} \quad \therefore \quad Adj(B^T) = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

So, B is symmetric

17.(A) We know that $\cos\alpha = \frac{L_1.L_2}{|L_1|.|L_2|}$ where α is the angle between the vectors L_1, L_2.

$\Rightarrow \quad \cos\alpha = \frac{\lambda\mu(a(\cos\theta + \sqrt{3}) + b(\sqrt{2}\sin\theta) + c(\cos\theta - \sqrt{3}))}{\lambda\mu(\sqrt{a^2 + b^2 + c^2}).(\sqrt{\cos^2\theta + 3 + 2\sqrt{3}\cos\theta + 2\sin^2\theta + \cos^2\theta + 3 - 2\sqrt{3}\cos\theta})}$

$\Rightarrow \quad \cos\alpha = \frac{(a + c)\cos\theta + \sqrt{3}(a - c) + b\sqrt{2}\sin\theta}{\sqrt{8}(\sqrt{a^2 + b^2 + c^2})}$

For α to be independent of θ

$a + c = 0$ … (i)

And $b = 0$ … (ii)

Therefore, $\cos\alpha = \frac{\sqrt{3}(a - c)}{\sqrt{8}.(\sqrt{a^2 + c^2})} \quad \Rightarrow \quad \cos\alpha = \frac{\sqrt{3}(2a)}{(\sqrt{8 \times 2a^2})}$

$\Rightarrow \quad \cos\alpha = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \alpha = \frac{\pi}{6}$

18.(C) Then from $15\left|\overrightarrow{AC}\right| = 3\left|\overrightarrow{AB}\right| = 5\left|\overrightarrow{AD}\right|; \quad \left|\overrightarrow{AB}\right| = 5\lambda$

Let θ be the angle between $\overrightarrow{BA}$ and $\overrightarrow{CD}$

$\Rightarrow \quad \cos\theta = \frac{\overrightarrow{BA}.\overrightarrow{CD}}{|\overrightarrow{BA}||\overrightarrow{CD}|} = \frac{-\vec{b}.(\vec{d} - \vec{c})}{|\vec{b}||\vec{d} - \vec{c}|}$ …(i)

Now $-\vec{b}.(\vec{d}-\vec{c})=\vec{b}.\vec{c}-\vec{b}.\vec{d}$ $\quad =|\vec{b}||\vec{c}|\cos\frac{\pi}{3}-|\vec{b}||\vec{d}|\cos\frac{2\pi}{3}$

$=(5\lambda)(\lambda)\frac{1}{2}+(5\lambda)(3\lambda)\frac{1}{2}$ $\quad =\frac{5\lambda^2+15\lambda^2}{2}$ $\quad =10\lambda^2$

Denominator of (i) $=|\vec{b}||\vec{d}-\vec{c}|$

Now $\quad |\vec{d}-\vec{c}|^2=|\vec{d}|^2+|\vec{c}|^2-2\vec{c}.\vec{d}$

$=9\lambda^2+\lambda^2-2(\lambda)(3\lambda)(1/2)$ $\quad =10\lambda^2-3\lambda^2$ $\quad =7\lambda^2$

Denominator of (i) $=(5\lambda)(\sqrt{7}\lambda)=5\sqrt{7}\lambda^2$ $\quad \therefore \quad \cos\theta=\frac{10\lambda^2}{5\sqrt{7}\lambda^2}=\frac{2}{\sqrt{7}}$

19.(A) We have $\cot\theta-\cot A=\cot B+\cot C$

Therefore $\frac{\sin(A-\theta)}{\sin A\sin\theta}=\frac{\sin(B+C)}{\sin B\sin C}=\frac{\sin A}{\sin B\sin C}$

$\frac{\sin(A-\theta)}{\sin\theta}=\frac{\sin^2 A}{\sin B\sin C}$ … (i)

Similarly, $\frac{\sin(B-\theta)}{\sin\theta}=\frac{\sin^2 B}{\sin C\sin A}$ … (ii)

and $\frac{\sin(C-\theta)}{\sin\theta}=\frac{\sin^2 C}{\sin A\sin B}$ … (iii)

Multiplying (i), (ii) & (iii) we get, $\frac{\sin(A-\theta)\sin(B-\theta)\sin(C-\theta)}{\sin^3\theta}=1$

20.(C) It is given that

$f(K)+a_K=\sum_{r=1}^{n}a_r$ and $\frac{1}{a_1},\frac{1}{a_2},....,\frac{1}{a_n}$ are in AP

Therefore $\frac{\sum_{r=1}^{n}a_r}{a_1},\frac{\sum_{r=1}^{n}a_r}{a_2},....,\frac{\sum_{r=1}^{n}a_r}{a_n}$ are in AP

$\frac{a_1+f(1)}{a_1},\frac{a_2+f(2)}{a_2},....,\frac{a_n+f(n)}{a_n}$ are in AP

$\frac{f(1)}{a_1},\frac{f(2)}{a_2},....,\frac{f(n)}{a_n}$ are in AP

Finally, $\frac{a_1}{f(1)},\frac{a_2}{f(2)},....,\frac{a_n}{f(n)}$ are in HP

SECTION-2

1.(1) $\because \quad (1+x+2x^2)^{20}=a_0+a_1x+a_2x^2+...+a_{40}x^{40}$

Put $x=1\Rightarrow 4^{20}=a_0+a_1+....+a_{40}$ …(i)

Put $x=-1\Rightarrow 2^{20}=a_0-a_1+....-a_{39}+a_{40}$ …(ii)

Subtract (ii) from (i), we get, $4^{20}-2^{20}=2(a_1+a_3+...+a_{37}+a_{39})$

$\Rightarrow \quad a_1 + a_3 + ... + a_{37} = 2^{39} - 2^{19} - a_{39}$ ...(iii)

$\because \quad a_{39} = \text{coeff of } x^{39} \text{ in } (1+x+2x^2)^{20}$

$= \frac{20!}{0!1!19!}(1)^{\circ}(2)^{19} = 20.2^{19}$

Substitute the value of a_{39} in equation (iii), we get

$a_1 + a_3 + ... + a_{37} = 2^{39} - 2^{19} - 20.2^{19}$

2.(1) We have

$1 = 3-2,\ 5 = 3^2 - 2^2,\ 19 = 3^3 - 2^3,\ 65 = 3^4 - 2^4,\ 211 = 3^5 - 2^5, ...$

Therefore sum to n terms is:

$$\sum_{K=1}^{n}(3^K - 2^K) = (3 + 3^2 + ... + 3^n) - (2 + 2^2 + ... + 2^n)$$

$$= \frac{3(3^n-1)}{3-1} - \frac{2(2^n-1)}{2-1} \qquad = \frac{3}{2}(3^n-1) - 2(2^n-1) \qquad = \frac{1}{2}(3^{n+1} - 2^{n+2} + 1)$$

3.(2) $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x} dx$

$I_n = \int_{-\pi}^{\pi} \frac{\pi^x \sin nx}{(1+\pi^x)\sin x} dx$

(by property $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$)

$2I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{\sin x} dx \Rightarrow 2I_n = 2\int_0^{\pi} \frac{\sin nx}{\sin x} dx; \qquad I_n = \int_0^{\pi} \frac{\sin nx}{\sin x} dx$

Now, $I_{n+2} - I_n = \int_0^{\pi} \frac{\sin(n+2)x - \sin nx}{\sin x} dx \qquad = \int_0^{\pi} \frac{2\cos(n+1)x \sin x}{\sin x} dx$

$$= 2\left[\frac{\sin(n+1)x}{(n+1)}\right]_0^{\pi} = 0 \qquad \Rightarrow \qquad I_{n+2} = I_n$$

4.(3094) Number of ways in which only one person get his own bag

$= {}^7C_1 \times$ (Derangement of 6 objects)

$= {}^7C_1 \times 6!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!}\right)$

$= 7 \times 265 = 1855$

Number of ways in which exactly two persons get their own bag

$= {}^7C_2 \times$ (Derangement of 5 objects)

$= {}^7C_2 \times 5!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right) \quad = 21 \times 44 \qquad = 924$

Numbers of ways in which exactly three persons get their own bag

$= {}^7C_3 \times$ (Derangement of 4 objects)

$= {}^7C_3 \times 4!\left(1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right) = 35\times 9 \qquad = 315$

$\therefore$ Required number ways = 1855 + 924 + 315 = 3094

5.(0) Let $L_1 : x = y = z, L_2 : x-1 = y-2 = z-3$ be two lines.

Let the foot of perpendicular to L_2 from origin O be A. Segment OA is rotated about O by $\frac{\pi}{2}$ such that L_2 rotates with it, without changing its direction cosines. If the new position of A is $B(\alpha,\beta,\gamma)$ then $\alpha+\beta+\gamma$ is:

Note $L_1 \| L_2$ and $OA \perp L_2$

If coordinates of $A(a_1,a_2,a_3)$ then $a_1+a_2+a_3 = 0$,

After rotation the new coordinates of $A \to B(\alpha,\beta,\gamma)$

OB will also be perpendicular to the line L_1

$\therefore \quad \alpha+\beta+\gamma = 0$

6.(41) The given expansion $\left(\sqrt{2}+3^{1/5}\right)^{10} = \sum_{r=0}^{10} {}^{10}C_r\, 2^{\frac{10-r}{2}}.3^{\frac{r}{5}}$

The rational terms correspond for $r = 0, r = 10$

Hence the sum of rational terms $= {}^{10}C_0 2^5 + {}^{10}C_{10} 3^2 = 32 + 9 = 41$

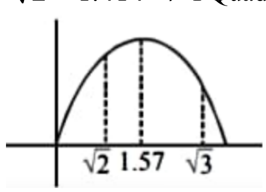
7.(2) Given that $f(x+y) = f(x).f(y)$ ...(i)

$f'(0) = \lim_{h\to\infty} \frac{f(h)-f(0)}{h} = 2$ ...(ii)

Now, $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ $f'(x)$; $\quad f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x+0)}{h}$

$= \lim_{h\to 0} \frac{f(x).f(h)-f(x).f(0)}{h} \qquad = \lim_{h\to 0} f(x)\frac{f(h)-f(0)}{h}; \qquad f'(x) = 2f(x)$

8.(3) $\sqrt{2} = 1.414 \Rightarrow$ I Quadrant



$\sqrt{3} = 1.732 \Rightarrow$ II Quadrant

$\left(\sin\sqrt{2} - \sin\sqrt{3}\right) \Rightarrow +ve$

$\left(\cos\sqrt{2} - \cos\sqrt{3}\right) \Rightarrow +ve$

$\Rightarrow$ ellipse & $\left(\cos\sqrt{2} - \cos\sqrt{3}\right) > \left(\sin\sqrt{2} - \sin\sqrt{3}\right)$

9.(1) $p_i = -\frac{b}{a}p_{i-1} - \frac{c}{a}p_{i-2} - \frac{d}{a}p_{i-3}$

$$p_{10} = -6p_9 - 4p_8 - \frac{5}{1}p_7$$

$$p_7 = \left|\frac{p_{10} + 6p_9 + 4p_8}{-5}\right|$$

10.(5) $M^{-1} = \begin{bmatrix} 3 & 4 & 5 \\ 4 & 5 & 3 \\ 5 & 3 & 4 \end{bmatrix}$

$\Rightarrow \quad |M^{-1}| = 3(20-9) - 4(16-15) + 5(12-25)$

$\Rightarrow \quad |M^{-1}| = -36$

And $tr(M^{-1}) = 3 + 5 + 4 = 12$

We know that $M^{-1} = \frac{adj(M)}{|M|}$

$\Rightarrow \quad adj(M) = |M|.M^{-1} \quad \Rightarrow \quad tr(adj(M)) = |M|\,tr(M^{-1})$

$\Rightarrow \quad tr(adj(M)) = \frac{-1}{36} \times 12 = -\frac{1}{3}; \quad |15tr(adj(M))| = \left|15 \times \left(-\frac{1}{3}\right)\right| = 5$